# REPORT 1206

# A REVISED GUST-LOAD FORMULA AND A RE-EVALUATION OF V-G DATA TAKEN ON CIVIL TRANSPORT AIRPLANES FROM 1933 TO 1950 [1]

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## SUMMARY

*A revised gust-load formula with a new gust factor is derived to replace the gust-load formula and alleviation factor widely used in gust studies. The revised formula utilizes the same principles and retains the same simple form of the original formula but provides a more appropriate and acceptable basis for gust-load calculations. The gust factor is calculated on the basis of a one-minus-cosine gust shape and is presented as a function of a mass-ratio parameter in contrast to the ramp gust shape and wing loading, respectively, used for the alleviation factor.*

*A summary of gust-velocity data from V-G records taken on civil transport airplanes from 1933 to 1950, re-evaluated by the revised formula is also presented. The results indicate that the conclusions drawn from previously presented data based on the original formula (in particular, concerning the levels of evaluated gust velocities between different routes) remain essentially unchanged. The National Advisory Committee for Aeronautics will make use of the revised gust-load formula in the evaluation of relevant gust data.*


## INTRODUCTION

A gust-load formula, embodying a number of simplifying assumptions, has long been used in this country for the calculation of design gust loads on ordinary airplanes by military and civilian regulating agencies (see, for example, ref. 1). This formula was developed and has been utilized by the NACA in the evaluation and interpretation of gust and gust-loads data obtained from measurements of accelerations and airspeeds experienced during routine and some special flights through turbulent air (see, for example, refs. 2 to 7). The formula may be written as

$$a_{n_{max}}=\frac{m\rho_0 S V_e U_e}{2W}K$$

where the quantities and customarily used units are as follows:

$a_{n_{max}}$ airplane maximum nondimensional normal acceleration, in $g$ units ($a_{n_{max}}=\Delta n_{max}$ in refs.)
$m$ wing lift-curve slope, per radian
$\rho_0$ air density at sea level, slugs/cu ft
$S$ wing area, sq ft
$V_e$ equivalent airspeed, fps
$U_e$ "effective" gust velocity, fps
$W$ airplane weight, lb
$K$ dimensionless "alleviation factor"

The formula serves to relate the peak accelerations due to gusts to be expected on a given airplane to the peak accelerations measured on another airplane for flight through the same rough air. The underlying concept is that a measured acceleration due to a gust may be used to derive an "effective" gust velocity which in turn is used to calculate the acceleration on another airplane by reversing the process. The effective gust velocity $U_e$ is not, therefore, a direct physical quantity but is rather a gust-load transfer factor definable in terms of the formula.

The nondimensional parameter $K$ depends on such factors as gust shape and resulting airplane motions. In order to allow for some of these factors and for simplicity in practical application, $K$ has been calculated on the basis that the gust shape is of a ramp type (gust velocity increasing linearly with distance up to a limit of 10 chords) and by taking into account effects of gust penetration and of the resulting vertical motion of the airplane. A small adjustment was then made to the parameter $K$ on the basis of model tests and analyses to allow for overall effects of pitching motion on the normal acceleration. The correction made implied that on all aircraft the acceleration is affected to about the same degree by the pitching motion, this assumption being reasonable only for conventional aircraft having satisfactory flying qualities. On this basis, $K$ is dependent only on a nondimensional mass-ratio parameter which is defined by the mass of the airplane divided by that of a cylinder of air about the wing. For design purposes, however, $K$ was expressed in terms of wing loading and was normalized by dividing by its value for $W/S=16$ lb/sq ft. This procedure had two effects which now can be considered undesirable. The use of wing loading rather than mass ratio ignored certain effects of altitude and airplane size, and the normalization produced effective gust velocities that are not referred directly to the maximum velocity of the ramp profile but rather to a constant times this value.

Over the years, the alleviation factor $K$ has been modified by the various regulating agencies in their design requirements. As a result, there now exist several different alleviation factors and correspondingly different design gust velocities. This situation has resulted in some confusion

[1] Supersedes NACA TN's 2964 by Kermit G. Pratt, 1953, and 3041 by Walter G. Walker, 1953.

when the design gust velocities used by the various agencies are compared with each other or with NACA gust data.

In order to provide for uniformity of gust-load calculations, the interested regulating agencies and the NACA, at a meeting of the ANC–1 Panel on Flight Loading Conditions, agreed to the desirability of adopting a new standard alleviation factor. This new alleviation factor, to be referred to as "gust factor," was to be based on the more fundamental parameter, mass ratio, instead of wing loading and also on a new gust profile represented by a one-minus-cosine curve.

The NACA agreed to calculate the new gust factor and to use it in a revised gust-load formula for the reduction of relevant gust data. A point of interest is that the new gust factor as calculated is not normalized to any given value, and hence the gust velocity can be conveniently referred directly to the maximum of the gust profile. Since the revised formula will be used in evaluating future NACA gust-research data, it appeared desirable to re-evaluate previously obtained acceleration and airspeed data from V–G records by the use of the revised formula. (The NACA V–G recorder is briefly described in ref. 2.) This report presents the revised gust-load formula and the new gust factor together with a summary of re-evaluated gust-velocity data from V–G records of civil transport airplanes for the period from 1933 to 1950. Most of the V-G data were originally presented in references 3 to 7.

## SYMBOLS

$A$ aspect ratio, $b^2/S$

$a_n$ nondimensional vertical or normal acceleration, $\frac{d^2z}{dt^2}\Big/g$, $g$ units

$a_{n_s}$ reference nondimensional vertical or normal acceleration, $\frac{m\rho SVU}{2W}$, $g$ units

NOTE.—As a result of a change in symbol standardization $a_n$ and $a_{n_s}$ replace herein $\Delta n$ and $\Delta n_s$, respectively, which are used in the references.

$b$ wing span, ft

$C_{L_g}(s)$ transient lift response to penetration of sharp-edge gust

$C_{L_\alpha}(s)$ transient lift response to unit-jump change in angle of attack

$c$ reference wing chord $\left(\text{mean geometric chord, } \frac{\text{Wing area}}{\text{Wing span}}, \text{ft}\right)$

NOTE.—In the present analysis, the choice of a reference chord is not critical. If preferred, the mean aerodynamic chord as defined by $\bar{c}=\frac{2}{S}\int_0^{b/2} c_y^2\,dy$, where $c_y$ is the local chord and $y$ is the distance along the span (ft), may be used. Whatever chord is selected as the reference chord should, of course, be used consistently for the purpose of data comparison.

$e$ base of the natural system of logarithms

$g$ acceleration due to gravity, ft/sec²

$H$ gust-gradient distance (horizontal distance from zero to maximum gust velocity), chords

$K$ alleviation factor defined in reference 2

$K_g$ gust factor (revised alleviation factor)

$l$ average flight miles to equal or exceed a given value of gust velocity

$m$ wing lift-curve slope, per radian

$M$ airplane mass, slugs

$N$ total number of observations in a sample of data

$P$ probability that the maximum value in a sample of data will equal or exceed a given value

$S$ wing area, sq ft

$s$ distance of penetration into gust, chords

$s_1$ dummy variable in superposition integral, chords

$t$ time, sec

$t_1$ dummy variable in superposition integral, sec

$U$ gust velocity (maximum value), fps

$U_{de}$ "derived" gust velocity, fps

$U_e$ effective gust velocity defined in reference 2, fps

$u$ gust velocity at any penetration distance, fps

$V$ airspeed, fps

$V_C$ design cruising speed, mph (ref. 1, p. 3)

$V_e$ equivalent airspeed, $V\sigma^{1/2}$, fps (see ref. 8)

$W$ airplane weight, lb

$z$ airplane vertical displacement (positive upward), ft

$\gamma$ location parameter of distribution of extreme values (symbol $u$ in ref. 9, p. 2)

$\lambda$ scale parameter of distribution of extreme values (symbol $\alpha$ in ref. 9, p. 2)

$\mu_g$ airplane mass ratio (sometimes referred to as "mass parameter" in the past), $\frac{2W}{m\rho cgS}$

$\rho$ air density, slugs/cu ft

$\rho_0$ air density at sea level, slugs/cu ft

$\sigma$ air-density ratio, $\rho/\rho_0$

$\tau$ average flight time per V–G record, hr

Subscript:

$max$ maximum value

A bar over a symbol denotes the mean value of the variable.

## REVISED GUST-LOAD FORMULA

### DERIVATION OF REVISED FORMULA

The revised gust-load formula to be derived herein, like the original formula, was obtained from solutions of an equation of airplane vertical motion in an isolated gust. The use of the formula to transfer accelerations from one airplane to another for continuous rough air implies, therefore, the assumption that the relative loads for single isolated gusts are a measure of the relative loads in a sequence of gusts. In regard to this assumption, it is recognized that some of the more recent methods for analysis of airplane loads in continuous rough air with proper allowance for various degrees of freedom of airplane motion may in due course be adopted; however, for the present, it remains desirable to retain the simplicity of the original method. As in the case of the original formula, the present method will not be suitable for all airplane configurations. Unusual airplanes will require special analysis. After the presentation of the revised gust-load formula, a brief comparison of features of the original and revised formulas is given.

**Basic assumptions and equation of motion.**—The equation of motion is based on the following assumptions commonly used in gust-load problems:

(1) The airplane is a rigid body.

(2) The airplane forward speed is constant.

(3) The airplane is in steady level flight prior to entry into the gust.

(4) The airplane can rise but cannot pitch.

(5) The lift increments of the fuselage and horizontal tail are negligible in comparison with the wing lift increment.

(6) The gust velocity is uniform across the wing span and is parallel to the vertical axis of the airplane at any instant.

If the forces associated with steady level flight are disregarded, a summation of vertical or normal forces on the airplane in a gust yields the following equation of motion:

$$M\frac{d^2z}{dt^2}+\frac{\rho}{2}V^2Sm\int_0^t\frac{1}{2\pi}C_{L_\alpha}(t-t_1)\frac{d^2z}{dt_1^2}\frac{1}{V}dt_1$$

$$=\frac{\rho}{2}V^2Sm\frac{U}{V}\int_0^t\frac{1}{2\pi}C_{L_g}(t-t_1)\frac{d\left[\frac{u(t_1)}{U}\right]}{dt_1}dt_1+\frac{\rho}{2}V^2Sm\frac{U}{V}\frac{u(0)}{U}\frac{1}{2\pi}C_{L_g}(t) \qquad (1)$$

In equation (1), the first term on the left-hand side is the inertia reaction and the second term is the damping force due to airplane vertical velocity. On the right-hand side, both terms are forces due to the gust; the first term is the force due to a gust having zero velocity at the beginning of penetration by the airplane and the second term is the force due to a gust having an initial velocity other than zero at the beginning of penetration.

By using the relationships $\frac{d^2z}{dt^2}=a_ng$ and $t=\frac{sc}{V}$, equation (1) can be written in nondimensional form as

$$\frac{a_n(s)}{a_{n_s}}+\frac{1}{\mu_g}\int_0^s\frac{1}{2\pi}C_{L_\alpha}(s-s_1)\frac{a_n(s_1)}{a_{n_s}}ds_1$$

$$=\int_0^s\frac{1}{2\pi}C_{L_g}(s-s_1)\frac{d\left[\frac{u(s_1)}{U}\right]}{ds_1}ds_1+\frac{u(0)}{U}\frac{1}{2\pi}C_{L_g}(s) \qquad (2)$$

where

$$a_{n_s}=\frac{m\rho SVU}{2W} \qquad (3)$$

$$\mu_g=\frac{2W}{m\rho cgS} \qquad (4)$$

and the functions $C_{L_g}$ and $C_{L_\alpha}$ are the transient lift responses of a wing to a penetration of a sharp-edge gust and to a unit-jump change in angle of attack, respectively. In equation (2), $a_n$ is the vertical acceleration that results from the gust and $a_{n_s}$ is a convenient reference acceleration which may be interpreted as the acceleration that would result solely from a lift force equal to the steady-state lift associated with the maximum velocity of the gust. The second term is associated with the damping due to the airplane vertical velocity and the remaining terms are associated directly with the gust. It can be remarked that the mass ratio $\mu_g$ is a basic parameter in equation (2).

**Solution of the equation of motion.**—Equation (2) was solved for histories of the acceleration ratio $a_n(s)/a_{n_s}$ on the basis of the following transient lift functions and gust shape.

The transient lift functions used are

$$\frac{1}{2\pi}C_{L_\alpha}(s)=1.000-0.165e^{-0.090s}-0.335e^{-0.600s} \qquad (5)$$

$$\frac{1}{2\pi}C_{L_g}(s)=1.000-0.236e^{-0.116s}-0.513e^{-0.728s}-0.171e^{-4.84s} \qquad (6)$$

These are the transient lift functions for infinite aspect ratio given in reference 10, normalized to asymptotic values of unity. These expressions, rather than finite-aspect-ratio functions (such as those given in ref. 10), were used for simplicity in order to provide solutions of the equation of motion independent of aspect ratio except, of course, as aspect ratio affects the slope of the lift curve. Thus, in effect, only the shapes of the infinite-aspect-ratio functions are used, the appropriate finite-aspect-ratio lift-curve slope being used in evaluating the mass ratio $\mu_g$. The results obtained through the use of equations (5) and (6), however, are probably less than 5 percent different from the results that would be obtained through the use of the finite-aspect-ratio functions, as indicated by some limited information in reference 11. This reference also indicates that the differences might be slightly larger when the transient lift functions for a Mach number of 0.7 are used.

The gust shape used was that designated by the ANC-1 Panel, that is,

$$\left.\begin{aligned}\frac{u(s)}{U}&=\frac{1}{2}\left(1-\cos\frac{\pi s}{H}\right)=\sin^2\frac{\pi s}{2H} && (0<s<2H)\\ \frac{u(s)}{U}&=0 && (0>s>2H)\end{aligned}\right\} \qquad (7)$$

where $H$ was designated equal to 12.5 chords. (Inasmuch as the initial portion of the revised gust profile is relatively ineffective, the gradient distance of 12.5 chords corresponds roughly to the 10-chord gradient distance for the original ramp profile.)

With these lift functions and gust shape, equation (2) is noted to depend on only one parameter, the mass ratio $\mu_g$. Solutions of the equation were obtained for a range of $\mu_g$, by the numerical recurrence method presented in reference 12 for the case of a rigid airplane. Although solutions of the equation also can be obtained in closed form when equations.

(5) and (6) are used, the numerical method was chosen because it is much more rapid, is easy to apply, and gives good accuracy (error in $a_n/a_{n_s}$ less than $\pm 0.005$). Sample histories of the calculated acceleration ratio for three different values of $\mu_g$ are presented in figure 1.

**Revised gust factor and gust-load formula.**—Since the maximum value of $a_n/a_{n_s}$ with respect to gust penetration distance (see fig. 1) defines the maximum acceleration experienced by the airplane, it is of primary concern in design. This maximum value is herein designated as the "gust factor" and is labeled $K_g$; that is,

$$\left(\frac{a_n}{a_{n_s}}\right)_{max}=K_g \tag{8}$$

The variation of this gust factor with mass ratio is shown in

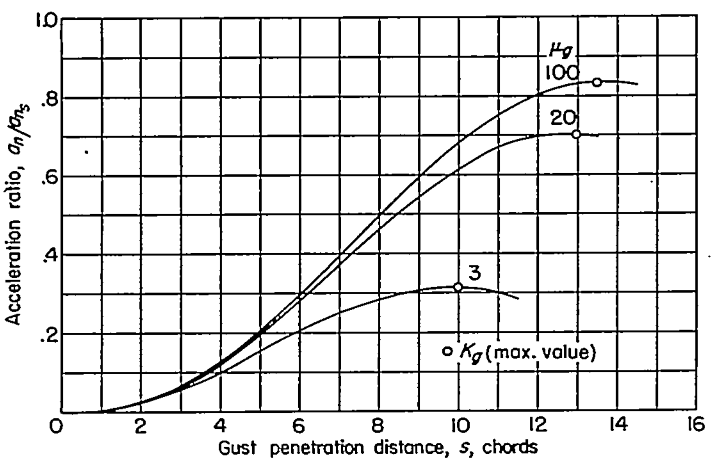



FIGURE 1.—Representative histories of acceleration ratio.

figure 2. No closed-form analytical expression for the curve of $K_g$ can be written, since it was obtained by a numerical procedure. A convenient expression which closely approximates the curve was found, however, and is presented below:

$$K_g=\frac{0.88\mu_g}{5.3+\mu_g} \tag{9}$$

This simple expression gives $K_g$ with an error less than $\pm 0.01$.

The revised gust-load formula follows directly from equation (8); that is,

$$a_{n_{max}}=a_{n_s}K_g$$

$$=\frac{m\rho SVU}{2W}K_g \tag{10}$$

In terms of equivalent speeds this equation becomes

$$a_{n_{max}}=\frac{m\rho_0 SV_e U_{de}}{2W}K_g \tag{11}$$

where the subscript $e$ is used to denote that both the airspeed and gust velocity are equivalent speeds. The subscript $d$ has been added also to the gust velocity to denote that, when the formula is used to evaluate gust velocities from measured accelerations, the gust velocities obtained, like $U_e$ in the original formula, are "derived" rather than measured values. For application in design, however, $U_{de}$ may of course be a stipulated value.

The revised gust-load formula (eq. (11)) may be noted to be of the same form as the original formula, the gust factor $K_g$ being in effect a revision of the alleviation factor $K$. A further comparison of the original and new formulas is given in the subsequent section.

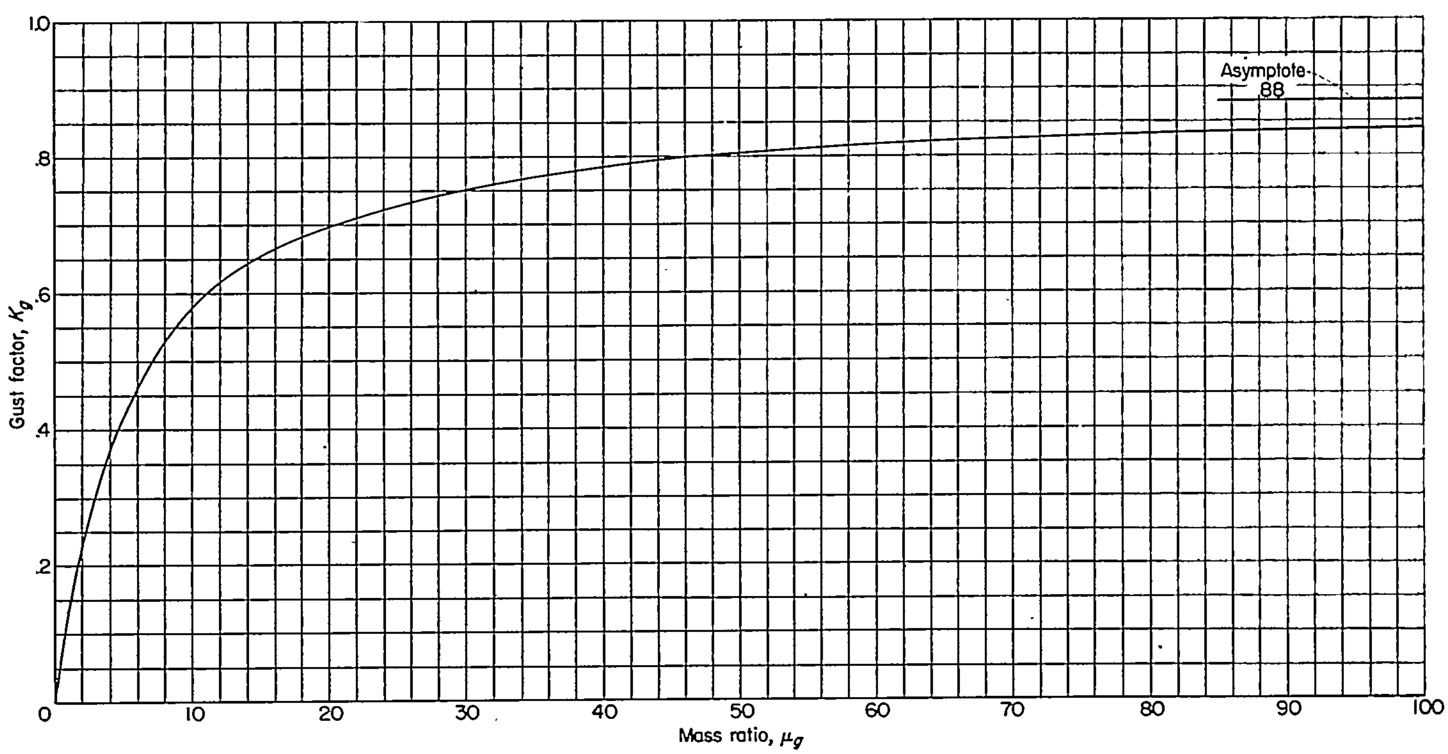
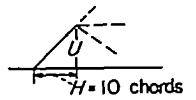
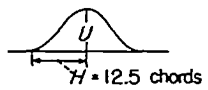
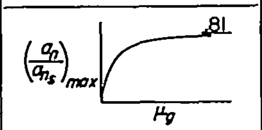
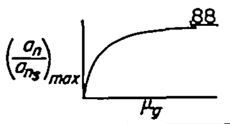
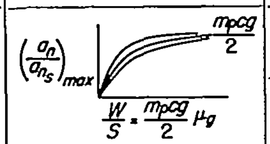
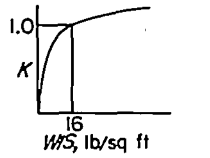
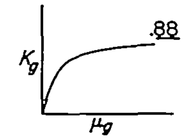



FIGURE 2.—Gust factor $K_g$ as a function of mass ratio $\mu_g$ for standard gust shape $\frac{u}{U}=\frac{1}{2}\left(1-\cos\frac{\pi s}{12.5}\right)$.

Revised gust-load formula, $a_{n_{max}}=\frac{m\rho_0 SV_e U_{de}}{2W}K_g$ where $\mu_g=\frac{2W}{m\rho cgS}$.

### COMPARISON OF ORIGINAL AND REVISED GUST-LOAD FORMULAS

The salient features of the revised gust-load formula as compared with those of the original formula are illustrated in the following table:

| Item | Original | Revised |
|---|---|---|
| (a) Gust-load formula | $a_{n_{max}}=\frac{m\rho_0 SV_eU_e}{2W}K$ | $a_{n_{max}}=\frac{m\rho_0 SV_eU_{de}}{2W}K_g$ |
| (b) Gust shape |   |   |
| (c) Maximum acceleration ratio in gust |   |   |
| (d) Maximum acceleration ratio plotted against wing loading (corrected for pitch effects) |   | |
| (e) Alleviation and gust factors |   |   |
| (f) Gust velocity | $U_e \propto U\sigma^{1/2}$ | $U_{de}=U\sigma^{1/2}$ |

The description of the original curve, in the table and in the text to follow, is schematic in nature and is not intended to be sufficiently detailed to permit reproduction of the curve.

As item (a) shows, the forms of the original and the revised formulas are the same. The respective gust shapes are shown as item (b). The original gust shape was of a ramp type but was effectively undefined beyond a gradient distance of 10 chords as a consequence of an approximation made in solving the equation of motion. This approximation made use of the value of the acceleration at a penetration distance of 10 chords as the maximum acceleration if the actual maximum did not occur within this distance. The revised gust profile, in comparison, is symmetrical in shape, finite in length, and has a gust gradient distance of 12.5 chords.

For item (c), curves of maximum acceleration ratio $\left(\frac{a_n}{a_{n_s}}\right)_{max}$ (ratio of the maximum acceleration in the gust to the reference acceleration $a_{n_s}$) associated with the respective gust shapes are given as a function of mass ratio. As previously mentioned, the original curve of alleviation factor described in reference 2 was not used in terms of mass ratio but rather in terms of the convenient design parameter, wing loading. This use implies a separation of the left-hand curve of item (c) into a family of curves involving the parameter $m\rho cg/2$ as indicated for item (d). The alleviation factor was obtained from this family as a single curve which was not, however, a particular curve of the family but was obtained from the entire family on the basis of various engineering considerations. These considerations included

(1) An assumed variation of wing chord with wing loading
(2) An allowance, based on experiment and analysis, for the effects of pitching motion, consisting of a constant percentage correction to the maximum acceleration ratio (that is, multiplication of $(a_n/a_{n_s})_{max}$ in item (d) by a constant factor)
(3) Normalization of the curve to unity at $W/S=16$ lb/sq ft. The alleviation-factor curve thus obtained is shown as item (e).

Although the use of the single alleviation-factor curve $K$ based on wing loading does not fully account for variations in the parameter $m\rho cg/2$, at the time of derivation it was considered representative of airplane design and operating practice. At the present time, however, the variations of $m\rho cg/2$ have increased to the point where a single curve based on wing loading cannot be considered representative. In the light of modern airplane practice, it is now desirable to revert to a single curve for the gust factor which is based on the less restrictive and more fundamental parameter, mass ratio. The gust-factor curve $K_g$ is shown on the right-hand side of item (e); it is the same as that in item (c).

The original formula, as indicated in reference 2, has been subject to scrutiny in the form of continuing experiments in regard to usefulness for conventional airplanes and in regard to the effects of various other factors not explicitly taken into account in its derivation. This background of experience can be carried over in the use of the revised formula as well. In the same vein, the allowance for effects of pitching motion made in the derivation of the alleviation factor but not explicitly taken into account in the derivation of the gust factor nevertheless can be included in the use of the gust factor. The pitch correction was not directly applied to the gust factor because it would cancel out of calculations relating the acceleration of one airplane to that of another airplane.

As mentioned earlier, in the use of the formulas to evaluate measured accelerations, the derived gust velocity $U_{de}$ and the effective gust velocity $U_e$ are both derived rather than measured quantities. They differ, however, as indicated by item (f), in that $U_{de}$ corresponds to the maximum equivalent velocity of the gust shape, whereas $U_e$ corresponds to only a fraction of the maximum equivalent velocity of the original gust shape. This fraction stems from the value used to normalize the alleviation-factor curve at $W/S=16$ lb/sq ft. There is no single constant proportional relationship between $U_{de}$ and $U_e$ for all airplanes because of their respective mass-ratio and wing-loading bases.

Subsequent sections of this report will be devoted to an application of the revised formula to some previously obtained and reported NACA gust data.

### SUMMARY OF RE-EVALUATED GUST-VELOCITY DATA FROM V-G RECORDS

A principal application of the original gust-load formula by the NACA has been to obtain effective gust velocities from normal acceleration and airspeed data of V–G records. Since, however, the revised formula will be used in evaluating future relevant NACA gust-research data, some previously reported V–G data evaluated by use of the original formula have been re-evaluated by use of the revised formula in order to place them on a comparable basis with future data. The re-evaluated data are summarized herein.

**Scope of data.**—Table I shows the scope of the V–G data collected from 1933 to 1950 as presented in references 3 to 7. In accordance with the procedures of these references, the data are grouped into three time intervals—1933 to 1941, 1941 to 1945, and 1945 to 1950—to denote the operations prior to, during, and after a wartime period. The type of airplane and the route are identified by combinations of a capital letter and a Roman numeral, such as A–I, B–II, and C–III. The airplanes and routes which correspond to those given in reference 3 are identified herein by the same combinations to facilitate comparisons of present and older results.

Table II gives the airplane characteristics used for evaluating the data. The values given either were obtained from the Civil Aeronautics Administration, the design manual of the airplane manufacturer, or were computed as indicated in the table.

TABLE I.—SCOPE OF V–G DATA ANALYZED IN AIRLINE OPERATIONS FROM 1933 TO 1950

| Airplane | | Routes flown | Dates of operation | Number of records analyzed | Av. flight hours per record, $\tau$ | Total record hours |
|---|---|---|---|---|---|---|
| | | Period from 1933 to 1941 | | | | |
| A | I | Newark-Seattle-Oakland | July 1933 to Apr. 1937 | 30 | 305 | 9,168 |
| B | II | Miami-Newark-Boston | June 1935 to Dec. 1940 | 18 | 367.5 | 6,615 |
| C | III | Miami-Buenos Aires | Apr. 1936 to Dec. 1939 | 117 | 95.1 | 11,124 |
| D | IV | San Francisco-Hawaii-Hong Kong | June 1936 to Dec. 1941 | 100 | 128.1 | 12,807 |
| E | I | Newark-Seattle-Oakland | July 1937 to Dec. 1941 | 15 | 645 | 9,691 |
| E | V | Boston-Newark-Los Angeles | Feb. 1937 to Oct. 1939 | 37 | 275 | 10,187 |
| E | VI | Newark-Kansas City-Los Angeles. | Sept. 1938 to Oct. 1940 | 11 | 295 | 3,232 |
| F | III | Caribbean region and northern part of South America. | Apr. 1940 to Dec. 1941 | 83 | 29 | 2,386 |
| | | Period from 1941 to 1945 | | | | |
| D | IV | San Francisco-Hawaii | Dec. 1941 to Jan. 1945 | 30 | 36.1 | 1,084 |
| E | I | Newark-Seattle-Oakland | Dec. 1941 to Dec. 1944 | 20 | 695 | 13,911 |
| F | III | Caribbean region and northern part of South America. | Dec. 1941 to Sept. 1944 | 193 | 53 | 10,261 |
| | | Period from 1945 to 1950 | | | | |
| E | VII | New Orleans-Kansas City-Minot, N. D. | Oct. 1948 to Feb. 1950 | 79 | 303 | 23,940 |
| G | II | New York-Miami | Nov. 1947 to Feb. 1950 | 194 | 248 | 48,187 |
| H | III | Miami-Caribbean region-South America. | Nov. 1947 to May 1949 | 27 | 247 | 6,677 |
| H | IV | San Francisco-Australia-Orient. | Aug. 1947 to Apr. 1949 | 69 | 231 | 15,951 |
| J | VIII | New York-Seattle | Dec. 1948 to Apr. 1950 | 388 | 99.4 | 38,578 |

**Application of gust formulas to V-G records.**—The record from the V–G recorder presents an envelope of the maximum positive and negative accelerations experienced as a function of the airspeed. The gust-velocity data published in references 3 to 7 were evaluated from V–G records by substituting into the original gust-load formula the accelerations and associated airspeeds read from each V–G record envelope. Only the maximum positive and negative values of the effective gust velocities evaluated from each record were selected for analysis. The original formula (given in the introduction to this report) as transposed for this purpose is

$$U_{e_{max}}=\frac{2a_nW}{m\rho_0SV_eK} \tag{12}$$

where $a_n$ and $V_e$ are the accelerations and associated airspeeds giving the maximum positive and negative effective gust velocities $U_{e_{max}}$ for each record. It should be noted that owing to the effect of airspeed the values of acceleration which are associated with the maximum effective gust velocities are not necessarily the overall maximum positive and negative accelerations observed on the V–G record but, rather, are particular points on the record envelope. The subscript, *max*, was therefore dropped from $a_n$ for this application.

The application of the revised formula to the evaluation of V-G records is the same as that of the original formula. From equation (11) the maximum derived gust velocities for a given record are

$$U_{de_{max}}=\frac{2a_nW}{m\rho_0SV_eK_g} \tag{13}$$

where again $a_n$ and $V_e$ are the accelerations and associated airspeeds giving the maximum positive and negative gust velocities.

**Method of re-evaluation and results.**—The method of converting the measurements of $U_{e_{max}}$ into terms of $U_{de_{max}}$ follows directly from the definitions of the two quantities. From equations (12) and (13)

$$U_{de_{max}}=U_{e_{max}}\frac{K}{K_g} \tag{14}$$

This relation permits simple conversion of the values of $U_{e_{max}}$ obtained from measurements from a given airplane to values of $U_{de_{max}}$. It might be noted that in calculating $U_{de_{max}}$ the effects of air density on the airplane response are included, since the value of $K_g$ depends upon the mass ratio

TABLE II.—AIRPLANE CHARACTERISTICS

| Airplane | Design gross wt., $W$, lb | Wing area, $S$, sq ft | Wing span, $b$, ft | Mean geometric chord, $c$, ft | Aspect ratio, $A$ | Design cruising speed, $V_C$, mph | Estimated operating altitude, ft | Mass ratio, $\mu_g$ (a) | Gust factor $K_g$ | Gust factor $K$ (b) | Gust factor $\frac{K}{K_g}$ | Slope of lift curve, computed from $m=\frac{6A}{A+2}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | 13,400 | 836 | 74 | 11.3 | 6.6 | 180 | 5,000 | 7.94 | 0.526 | 0.960 | 1.83 | 4.60 |
| B | 18,560 | 939 | 85 | 11.0 | 7.7 | 215 | 5,000 | 9.75 | .570 | 1.008 | 1.77 | 4.76 |
| C | 41,000 | 1,340 | 118.2 | 11.3 | 10.4 | 181 | 5,000 | 13.85 | .637 | 1.098 | 1.73 | 5.04 |
| D | 50,000 | 2,145 | 130 | 16.5 | 7.9 | 168 | 5,000 | 7.62 | .518 | 1.045 | 2.02 | 4.78 |
| E | 25,200 | 987 | 95 | 10.4 | 9.1 | 211 | 5,000 | 12.85 | .621 | 1.064 | 1.71 | 4.92 |
| F | 45,000 | 1,486 | 107.3 | 13.9 | 7.8 | 230 | 5,000 | 11.75 | .610 | 1.097 | 1.80 | 4.78 |
| G | 94,000 | 1,650 | 123 | 14.7 | 9.2 | 271 | 10,000 | 23.60 | .725 | 1.190 | 1.64 | 4.93 |
| H | 70,700 | 1,461 | 117.5 | 13.6 | 9.5 | 224 | 10,000 | 21.57 | .711 | 1.166 | 1.64 | 4.96 |
| J | 39,900 | 864 | 93.3 | 10.1 | 10.1 | 256 | 5,000 | 23.58 | .725 | 1.160 | 1.60 | 5.00 |

(a) For 0.85 gross weight at estimated operating altitude.
(b) For 0.85 gross weight.

which in turn is a function of air density. For the present calculations (as was done in ref. 3 and refs. 5 to 7), an operating weight was assumed equal to 85 percent of the airplane weight and a lift-curve slope was computed by using the relation $m=\frac{6A}{A+2}$ as indicated in table II. (Gust velocities are not given in reference 4; therefore, the normal-acceleration and airspeed data upon which that paper is based were re-evaluated to obtain values of $U_{e_{max}}$ and $U_{de_{max}}$ for this report.) Inasmuch as V-G records do not indicate the altitudes flown, it was necessary to estimate average operating altitudes from information received from the operator and from analysis of time-history data obtained on the airlines. The values of $K/K_g$ obtained for the various airplanes range from about 1.6 to 2.0 and are given in table II.

The application of equation (14) to the individual values of $U_{e_{max}}$ used to obtain the distributions of references 3 to 7 yielded values of $U_{de_{max}}$ for each of the airplanes listed in table I. The results are summarized in table III as frequency distributions of $U_{de_{max}}$. These $U_{de_{max}}$ distributions were then fitted with theoretical extreme-value distributions (see ref. 9) in order to smooth out the irregularities of the observed distributions and to provide a consistent basis for their extrapolation. The extreme-value distributions were fitted in accordance with the method of reference 9 by making use of the values of location parameter $\gamma$ and scale parameter $\lambda$ given in table III for each particular case. The theoretical distributions were then expressed as distributions of the probability $P$ of exceeding a given level of $U_{de_{max}}$ and, for convenience in comparing the various distributions, the probabilities were converted to flight distance by using the relation

$$l=\frac{0.8V_C\tau}{P} \tag{15}$$

In this relation $l$ is the average number of flight miles required to exceed given values of $U_{de_{max}}$, $\tau$ is the average flight time in hours per record for the respective data sample, and

TABLE III.—FREQUENCY DISTRIBUTIONS AND STATISTICAL PARAMETERS OF $U_{de_{max}}$

(a) Period 1933 to 1941

| $U_{de_{max}}$, fps | Number of observations for airplane and route | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | A-I | B-II | C-III | D-IV | E-I | E-V | E-VI | F-III |
| 4 to 8 | ------ | ------ | ------ | 2 | ------ | ------ | ------ | ------ |
| 8 to 12 | ------ | ------ | 7 | 9 | ------ | ------ | ------ | 3 |
| 12 to 16 | 1 | 2 | 20 | 32 | ------ | 2 | 5 | 33 |
| 16 to 20 | 1 | 7 | 28 | 40 | 2 | 1 | 0 | 46 |
| 20 to 24 | 5 | 7 | 65 | 35 | 3 | 9 | 6 | 30 |
| 24 to 28 | 3 | 5 | 57 | 34 | 7 | 14 | 4 | 29 |
| 28 to 32 | 8 | 7 | 27 | 21 | 3 | 16 | 4 | 13 |
| 32 to 36 | 12 | 3 | 19 | 16 | 8 | 9 | 2 | 7 |
| 36 to 40 | 11 | 3 | 5 | 2 | 4 | 10 | 0 | 1 |
| 40 to 44 | 7 | 0 | 2 | 4 | 1 | 5 | 1 | 3 |
| 44 to 48 | 6 | 1 | 2 | 0 | 2 | 5 | ------ | 1 |
| 48 to 52 | 2 | 1 | 2 | 3 | ------ | 0 | ------ | ------ |
| 52 to 56 | 1 | ------ | ------ | 1 | ------ | 1 | ------ | ------ |
| 56 to 60 | 2 | ------ | ------ | 0 | ------ | 2 | ------ | ------ |
| 60 to 64 | 1 | ------ | ------ | 0 | ------ | ------ | ------ | ------ |
| 64 to 68 | ------ | ------ | ------ | 0 | ------ | ------ | ------ | ------ |
| 68 to 72 | ------ | ------ | ------ | 1 | ------ | ------ | ------ | ------ |
| Total, N | 60 | 36 | 234 | 200 | 30 | 74 | 22 | 166 |
| $\overline{U}_{de_{max}}$, fps | 36.46 | 26.66 | 24.30 | 23.08 | 31.06 | 32.27 | 24.36 | 21.51 |
| $\gamma$ | 32.10 | 22.81 | 21.11 | 19.02 | 27.72 | 28.18 | 20.95 | 18.46 |
| $\lambda$ | 0.13 | 0.15 | 0.18 | 0.14 | 0.17 | 0.14 | 0.17 | 0.19 |

TABLE III.—FREQUENCY DISTRIBUTIONS AND STATISTICAL PARAMETERS OF $U_{de_{max}}$—Continued

(b) Period 1941 to 1945

| $U_{de_{max}}$, fps | Number of observations for airplane and route | | |
|---|---|---|---|
| | D-IV | E-I | F-III |
| 4 to 8 | ---- | ---- | 1 |
| 8 to 12 | 3 | ---- | 1 |
| 12 to 16 | 2 | ---- | 12 |
| 16 to 20 | 8 | ---- | 27 |
| 20 to 24 | 11 | 2 | 61 |
| 24 to 28 | 19 | 10 | 72 |
| 28 to 32 | 10 | 4 | 81 |
| 32 to 36 | 4 | 10 | 57 |
| 36 to 40 | 1 | 6 | 38 |
| 40 to 44 | 0 | 2 | 14 |
| 44 to 48 | 1 | 0 | 9 |
| 48 to 52 | 0 | 0 | 5 |
| 52 to 56 | 0 | 4 | 3 |
| 56 to 60 | 1 | 0 | 1 |
| 60 to 64 | ---- | 0 | 0 |
| 64 to 68 | ---- | 0 | 1 |
| 68 to 72 | ---- | 0 | 1 |
| 72 to 76 | ---- | 1 | 2 |
| 76 to 80 | ---- | 0 | ---- |
| 80 to 84 | ---- | 0 | ---- |
| 84 to 88 | ---- | 1 | ---- |
| Total, $N$ | 60 | 40 | 386 |
| $\overline{U}_{de_{max}}$, fps | 25.26 | 36.30 | 29.67 |
| $\gamma$ | 21.70 | 30.30 | 25.58 |
| $\lambda$ | 0.16 | 0.10 | 0.14 |

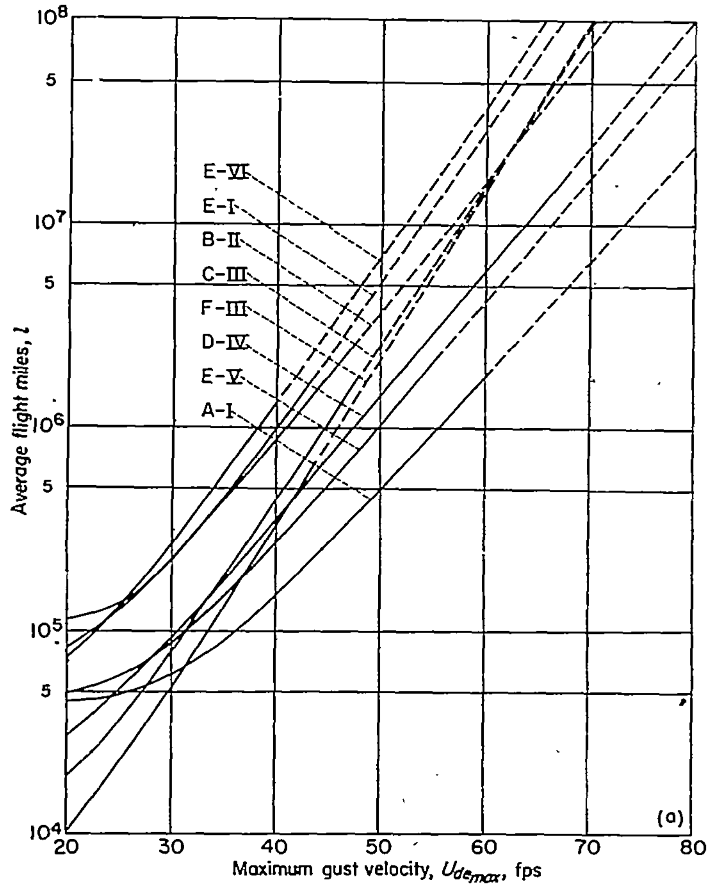
TABLE III.—FREQUENCY DISTRIBUTIONS AND STATISTICAL PARAMETERS OF $U_{de_{max}}$—Concluded

(c) Period 1945 to 1950

| $U_{de_{max}}$, fps | Number of observations for airplane and route | | | | |
|---|---|---|---|---|---|
| | E-VII | G-II | H-III | H-IV | J-VIII |
| 12 to 16 | -------- | 3 | -------- | 2 | -------- |
| 16 to 20 | 5 | 12 | 3 | 20 | 1 |
| 20 to 24 | 8 | 30 | 5 | 17 | 18 |
| 24 to 28 | 24 | 74 | 15 | 50 | 82 |
| 28 to 32 | 24 | 53 | 7 | 18 | 133 |
| 32 to 36 | 33 | 62 | 4 | 15 | 203 |
| 36 to 40 | 24 | 67 | 7 | 10 | 140 |
| 40 to 44 | 12 | 26 | 6 | 5 | 86 |
| 44 to 48 | 9 | 22 | 4 | 1 | 49 |
| 48 to 52 | 10 | 9 | 2 | -------- | 26 |
| 52 to 56 | 3 | 18 | 1 | -------- | 16 |
| 56 to 60 | 2 | 6 | -------- | -------- | 11 |
| 60 to 64 | 1 | 2 | -------- | -------- | 4 |
| 64 to 68 | 1 | 0 | -------- | -------- | 3 |
| 68 to 72 | 2 | 1 | -------- | -------- | 2 |
| 72 to 76 | -------- | 1 | -------- | -------- | 0 |
| 76 to 80 | -------- | 1 | -------- | -------- | 1 |
| 80 to 84 | -------- | 0 | -------- | -------- | 1 |
| 84 to 88 | -------- | 0 | -------- | -------- | -------- |
| 88 to 92 | -------- | 1 | -------- | -------- | -------- |
| Total, $N$ | 158 | 388 | 54 | 138 | 776 |
| $\overline{U}_{de_{max}}$, fps | 35.49 | 34.52 | 32.52 | 27.74 | 36.31 |
| $\gamma$ | 30.98 | 29.80 | 28.40 | 24.97 | 32.58 |
| $\lambda$ | 0.13 | 0.12 | 0.14 | 0.21 | 0.16 |

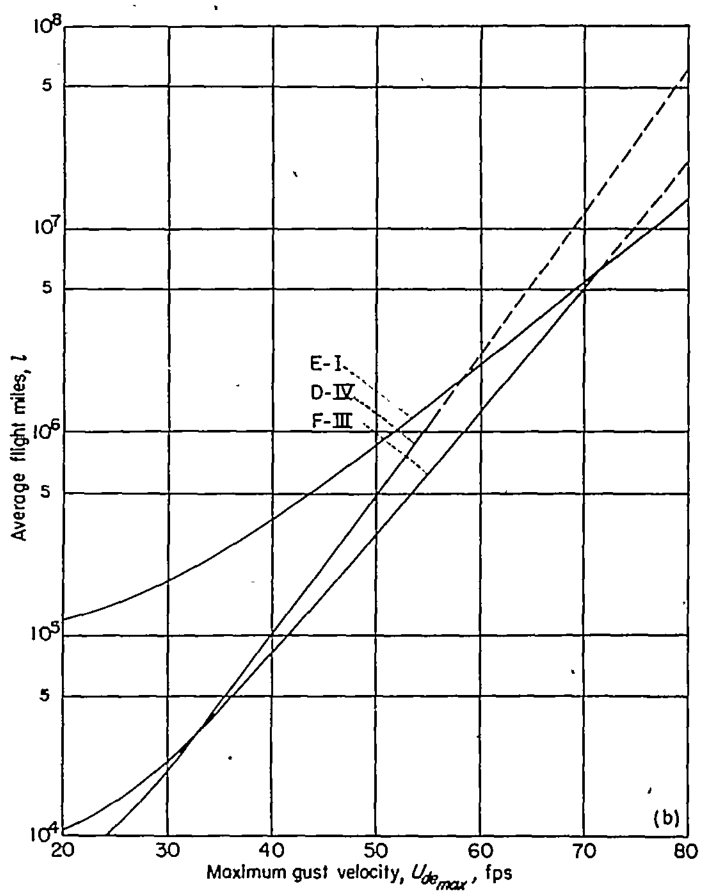
$0.8V_C$ is an assumed average operating airspeed in miles per hour. The results obtained by the application of equation (15) to the present data are shown in figure 3 (in three parts corresponding to the division of the data into the prewar, wartime, and postwar periods) which summarizes the gust velocities encountered in the various operations. The dashed portions of the curves indicate extrapolations beyond the limits of the data.

As a simple comparison of the levels of the gust velocities encountered in the various operations, the expected largest values of $U_{de_{max}}$ at $10^7$ flight miles were obtained from figure 3 and are listed in table IV. The corresponding values of $U_{e_{max}}$ obtained from the data in figure 2 of reference 3 for




(a) Period 1933 to 1941.

FIGURE 3.—Average flight miles $l$ for a maximum positive and negative gust velocity to equal or exceed a given value.




(b) Period 1941 to 1945.

FIGURE 3.— Continued.

$10^7$ flight miles are also given for comparison. (The values of $U_{de_{max}}$ shown in table IV differ in some cases from those obtained by scaling the values of $U_{e_{max}}$ by using equation (14). These differences are small and are mainly the result of minor differences in grouping and in curve-fitting methods used.)

In general, the levels of $U_{de_{max}}$ for the various operations remain essentially the same as the corresponding levels of $U_{e_{max}}$. The findings previously reported in reference 3—that the gusts experienced during the operations of these airplanes were largely independent of route, airplane, and operator—are not changed significantly.

## CONCLUDING REMARKS

A revised gust-load formula with a new alleviation factor termed "gust factor" has been derived herein to replace the gust-load formula widely used for design and gust studies. The revised formula, which is similar in form to the original formula, will be used by the NACA in the evaluation of relevant gust data. A brief comparison of the features of the two formulas has also been presented.

The revised gust-load formula has been used to re-evaluate the gust-velocity data computed from V–G records taken on civil transport airplanes during the period from 1933 to 1950, and the results have been summarized. The re-evaluation was made in terms of a "derived" gust velocity $U_{de}$, which is related to the "effective" gust velocity $U_e$ by a conversion factor that is a function of the type of airplane and the operating altitude. Although the value of the conversion factor varies from about 1.6 to 2.0 for the data presented, the conclusions drawn from the previously presented data based on $U_e$ (in particular, concerning the levels of evaluated gust velocities between different routes) remain essentially unchanged.

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
LANGLEY FIELD, VA., *September 8, 1953.*

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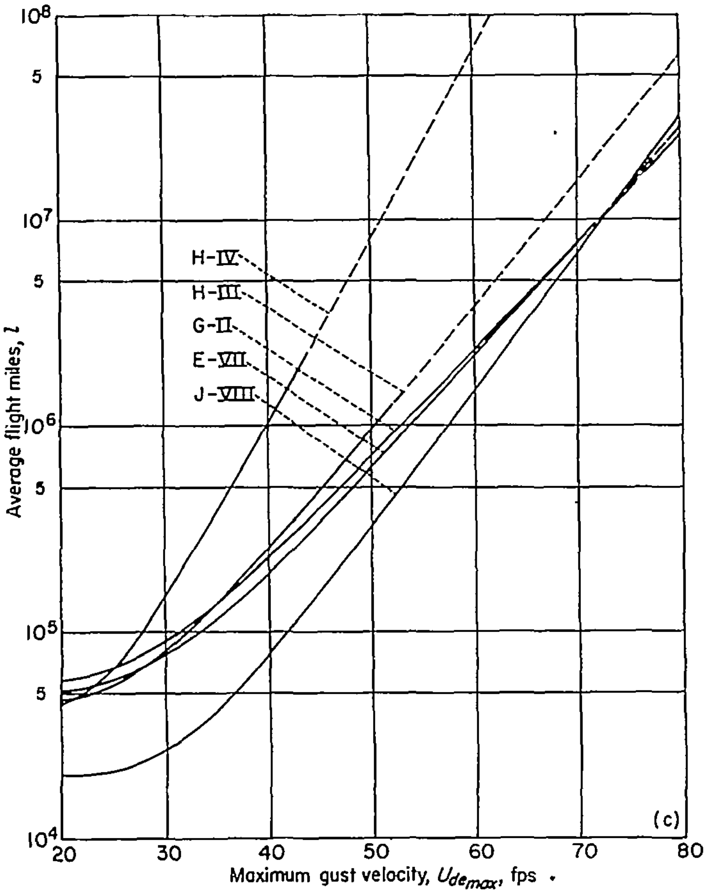


(c) Period 1945 to 1950.

FIGURE 3.—Concluded.

TABLE IV.—VALUES OF $U_{de_{max}}$ AND $U_{e_{max}}$ AT $10^7$ FLIGHT MILES

| Airplane and route | $U_{de_{max}}$ | $U_{e_{max}}$ |
|---|---|---|
| Period 1933 to 1941 | | |
| A-I | 73.3 | 42.1 |
| B-II | 56.5 | 33.4 |
| C-III | 57.5 | 33.7 |
| D-IV | 63.8 | 31.1 |
| E-I | 53.8 | 31.4 |
| E-V | 66.3 | 37.9 |
| E-VI | 52.2 | 33.8 |
| F-III | 58.1 | 32.6 |
| Period 1941 to 1945 | | |
| D-IV | 68.8 | 35.2 |
| E-I | 76.5 | 47.3 |
| F-III | 74.7 | 42.8 |
| Period 1945 to 1950 | | |
| E-VII | 72.2 | 42.2 |
| G-II | 72.4 | 43.4 |
| H-III | 67.1 | 39.9 |
| H-IV | 51.2 | 32.0 |
| J-VIII | 72.6 | 45.5 |

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